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THE CONDUCTANCE IN THE ONE DIMENSIONAL SPIN POLARIZED GAS

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Abstract

We present a theoretical analysis of the recent experimental results of Thomas *et al* on transport properties of spin polarized quantum wires. We suggest an explanation of the observed deviations of the conductance from the universal value $G = 2e^2/h$ per channel in the wire. We argue that the new quasi plateau observed for the conductance at the value $G = 1.4e^2/h$ is a result of the proximity between the spin polarized phase and the metallic one. The enhancement of the conductance from the value $G = e^2/h$ to $G = 1.4e^2/h$ is due to the hybridization of the electronic state at $K_F^{\downarrow} \approx 0$ with the chiral states at $K_F^{\uparrow}$ and $-K_F^{\uparrow}$.

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Recent experiments on quantum wires show discrepancies between the ballistic conductance measured by different groups [1–3]. According to the Tomonaga-Luttinger theory the ballistic conductance is normalized by the electron-electron interaction $G = K_c 2e^2/h$, where $K_c < 1$ for $\omega < v_c/L$ and $K_c = 1$ for $\omega > v_c/L$ (ω is the frequency, v_c is the charge density velocity and L is length of the wire). Contrary to these results, Shimizu [5] has argued that within the Landauer [6] theory the conductance remains unrenormalized. This result was understood by Finkelstein [7] and Kavabata [8] as a renormalization of both the current and the driven voltage. It is important to mention that the experimental and theoretical work has been restricted to cases where complete charge- spin separation exists. The Cavendish group experiment [3] is new in the sense that it raises a new question, namely ballistic transport in the presence of weak coupling between charge and spin excitations. We offer an explanation to this experiment and analyze for the first time the conductance in the presence of spin charge coupling.

The main features of the experiment are:

a)At low temperatures as the gate voltage is varied, the conductance shows a number of plateaus separated by the steps $G = 2e^2/h$. In addition to those plateaus, a new quasi plateau with the value of the conductance $G = 0.7 \times 2e^2/h$ is observed.

b)With the increase of the external magnetic field, the new quasi plateau shifts to lower values approaching $G = e^2/h$. This corresponds to the conductance of a single polarized channel.

c)As the temperature increases from 0.07 K to 1.5 K, the quasi plateau value of the conductance decreases.

d)The lack of inversion symmetry and the presence of interface electric field, induce zero field spin splitting in GaAs/AlGaAs heterostuctures. The splitting leads to the lift of the spin degeneracy and creates spin polarization sub-bands in a zero magnetic field.

Based on the experimental facts, we suggest the following explanation:

The two dimensional GaAs electron gas is polarized at T=0. Due to the fact that the GaAs electron mass is small $m = 0.08 m_e$ and $g = 0.5$, it follows that in the presence of a

magnetic field the Zeeman term is negligible with respect to the orbital motion. The lack of inversion symmetry allows the spin orbit coupling which lifts the spin degeneracy. As a result, the spin split is much larger than the Zeeman term would give. The presence of the orbital motion induces the coupling between the one dimensional modes when the electron motion is squeezed by a transversal electrostatic potential. Effectively we describe the spin polarization in the quantum wire by a field h, where $h = g\mu B + R\left(N^{\uparrow} - N^{\downarrow}\right)/\left(N^{\uparrow} + N^{\downarrow}\right)$. The second part of h includes the two dimensional many body effect which induces spin polarization at $T = 0$, so R is, in fact, an effective exchange constant. At low gate voltages only one mode is propagating. Due to the spin polarization only one component spin (spin up) is propagating, leading to the conductance of one channel, $G = e^2/h$. This indeed is seen in the experiment at large magnetic fields. When the magnetic field decreases, the polarization gap is reduced. Due to electron-electron interaction the propagating mode (spin up) is hybridized with the spin down mode. This enhances the conductance. We suggest that the quasi plateau at $G = 1.4 \times e^2/h$ occurs when the gate voltage is such that the propagating mode with the Fermi surface at $\pm K_F^{\uparrow} \neq 0$ is degenerated with the state $K_F^{\downarrow} = 0$. Increasing further the gate voltage, we obtain two propagating modes with four Fermi surfaces at $\pm K_F^{\uparrow} \neq 0$ and $\pm K_F^{\downarrow} \neq 0$. In this case the system behaves like a charged Luttinger liquid weakly coupled to the spin liquid. As a result we find that the conductance is $G = K_{\text{eff}} \times 2e^2/h$ with $K_{\text{eff}} \sim K_c$ (K_{eff} is renormalized by the spin liquid). If the arguments presented in ref. [5,7,8] hold for our case, we would expect to find $G = 2e^2/h$.

Formally we show this within the Hubbard model in the presence of a fixed magnetic field h and a tunable chemical potential E_F (by the gate voltage).

In the presence of the magnetic field h we identify the following cases:

a) The two (spin up and spin down) one dimensional modes are propagating. The value of the Fermi Surface points are given by the solution $\epsilon\left(\pm K_F^{\uparrow}\right) - E_F - h/2 = 0$ and $\epsilon\left(\pm K_F^{\downarrow}\right) - E_F + h/2 = 0$ with $K_F^{\uparrow} \neq 0$ and $K_F^{\downarrow} \neq 0$.

b)The polarized case, $\epsilon\left(\pm K_F^{\uparrow}\right) - E_F - h/2 = 0$ and no Fermi surface for the spin down band.

c)The phase transition case, characterized by the degeneracy of the three Fermi surfaces: $K_F^{\downarrow} = 0$, $+K_F^{\uparrow} \neq 0$ and $-K_F^{\uparrow} \neq 0$.

Let us consider the cases in details.

A. Two propagating modes – Isotropic case

We consider the one dimensional Hubbard model in the presence of a weak magnetic field h. Within the one dimensional Bosonization we find a weakly coupled charge and spin liquid. The charge is described by the bosonic field θ_c, canonical momentum P_c, charge density velocity v_c and charge stiffness $K_c < 1$. The spin liquid is described by the bosonic field θ_s, canonical momentum P_s, spin density velocity v_s and spin stiffness $K_s > 1$.

$$H = \int dx \left\{ \frac{v_c}{2} \left[K_c P_c^2 + \frac{(\partial_x \theta_c)^2}{K_c} \right] + \frac{v_s}{2} \left[K_s P_s^2 + \frac{(\partial_x \theta_s)^2}{K_s} \right] \right.$$
$$\left. + h \left[P_c P_s + \partial_x \theta_c \cdot \partial_x \theta_s \right] + \sqrt{\frac{2}{\pi}} V^{\mathrm{ext}}(x,t) \partial_x \theta_c \right\} \tag{1}$$

where $v_c = v_F(1 + g/\pi v_F)^{1/2}$, $v_s = v_F(1 - g/\pi v_F)^{1/2}$, $K_c = (1 + g/\pi v_F)^{-1/2}$, $K_s = (1 - g/\pi v_F)^{-1/2}$, $v_F^{\uparrow} = v_F + h$, $v_F^{\downarrow} = v_F - h$, $v_F = K_F/m$. The potential $V^{\mathrm{ext}}(x,t)$ is the external scalar potential introduced to probe the system. The effect of the weak magnetic field gives rise to a spin charge coupling. The conductance for this case will be $G = K_{\mathrm{eff}} \times 2e^2/h$. If the arguments given in ref. [5,7,8] will hold here, we expect to find $G = 2e^2/h$, in agreement with the plateau observed in the experiment.

B. Anisotropic case

When the electrons are polarized, only one mode is propagating, $C_{\uparrow}(x) = C(x)$. The second mode $C_{\downarrow}(x) = \psi(x)$ is characterized by the spin gap $D > 0$, $D = h/2 - E_F$. Increasing the gate voltage the spin gap vanishes inducing an enhancement of the conductance. When $D < 0$ we have the anisotropic case with $K_F^{\uparrow} \gg K_F^{\downarrow}$. The Hamiltonian is:

$$H=\int dx\left\{C^{\dagger}(x)\left(-\frac{\partial_x^2}{2m}-E_F-\frac{h}{2}\right)C(x)+\psi^{\dagger}(x)\left(-\frac{\partial_x^2}{2m}-E_F+\frac{h}{2}\right)\psi(x)\right.$$
$$\left.+g:c^{\dagger}(x)c(x):\psi^{\dagger}(x)\psi(x)+V^{\text{ext}}(x,t)\left(:C^{\dagger}(x)C(x):+\psi^{\dagger}(x)\psi(x)\right)\right\}, \tag{2}$$

where g is the Hubbard interaction and $V^{\text{ext}}(x,t)$ is the external potential.

We bosonize the metallic electrons:

$$C(x)=\frac{1}{\sqrt{2\pi a}}\left[e^{iK_F^{\uparrow}x}:e^{i\sqrt{4\pi}\theta_+}:+e^{-iK_F^{\uparrow}x}:e^{-i\sqrt{4\pi}\theta_-}:\right] \tag{3}$$

where $\theta_+ + \theta_- = \theta,\ \theta_- + \theta_+ = \phi,\ v_F^{\uparrow} = v_F + h,\ v_F = K_F/m$. The bosonic density $:C^{\dagger}(x)C(x):=1/\sqrt{\pi}\partial_x\theta+\frac{1}{\pi a}\cos\left(2K_F^{\uparrow}x+\sqrt{4\pi}\theta(x)\right)$. The Euclidean action S for the Hamiltonian (1) is

$$S=\int_0^{\beta}d\tau\int dx\left\{\frac{1}{2}\left[\frac{1}{v_F^{\uparrow}}\left(\partial_\tau\theta\right)^2+v_F^{\uparrow}\left(\partial_x\theta\right)^2\right]-\frac{\partial_x\theta}{\sqrt{\pi}}\left[g\psi^{\dagger}(x)\psi(x)-iV^{\text{ext}}(x,\tau)\right]\right.$$
$$\left.+\psi^{\dagger}(x,\tau)\left[\left(\partial_\tau-E_F+\frac{h}{2}+iV^{\text{ext}}(x,\tau)\right)-\frac{\partial_x^2}{2m}\right]\psi(x,\tau)\right\} \tag{4}$$

In the Eq. (4) we have ignored the oscillatory term $\cos\left(2K_F^{\dagger}x+\sqrt{4\pi}\theta(x)\right)$. We compute the generating function $W\left(V(x,t)\right)$ which will be used to compute the current.

$$Z=\int D\psi^{\dagger}D\psi D\theta\exp\{-S\}=\int D\psi^{\dagger}D\psi Da_0\exp\{-\tilde{S}\}=\exp\left\{-W\left(V^{\text{ext}}\right)\right\} \tag{5}$$

with

$$\tilde{S}=\tilde{S}\left(V^{\text{ext}}\right)+\tilde{S}\left(\psi^{\dagger},\psi\right)+\tilde{S}\left(a_0\right). \tag{6}$$

The integration of the bosonic variables θ induces an effective interaction $\tilde{U}(q,\omega_n)$ between the $\psi(x)=C_{\downarrow}(x)$ fermions. This induced two body interaction is replaced by the action $\tilde{S}\left(a_0\right)$ with the auxiliary scalar field a_0. The action in the Eq. (6) was obtained after the bosonic field θ was integrated . Using the Matsubara frequencies $\omega_n=2\pi Tn$ for the bosons and $\nu_n=2\pi T(n+1/2)$ for the fermions we obtain:

$$\tilde{S}\left(V^{\text{ext}}\right)=\sum_{\omega_n}\sum_q\frac{1}{2\pi v_F^{\uparrow}}V^{\text{ext}}\left(q,\omega_n\right)\frac{\left(v_F^{\uparrow}q\right)^2}{\left(v_F^{\uparrow}q\right)^2+\omega_n^2}V^{\text{ext}}\left(-q,-\omega_n\right) \tag{7}$$

$$\tilde{S}\left(\psi^{\dagger},\psi\right)=\sum_{\nu_m}\sum_{p}\left\{-\psi^{\dagger}\left(p,\nu_n\right)\left[-\nu_n-\frac{p^2}{2m}+\left(E_F+\frac{g^2}{2\pi v_F^{\uparrow}}-\frac{h}{2}\right)\right]\psi\left(p,\nu_n\right)\right.$$

$$\left.+i\sum_{\omega_n}\sum_{q}\psi^{\dagger}\left(p+q,\nu_n+\omega_n\right)\left[V^{\text{ext}}\left(q,\omega_n\right)\left(1-\frac{g}{\pi v_F^{\uparrow}}\frac{\left(v_F^{\uparrow}q\right)^2}{\left(v_F^{\uparrow}q\right)^2+\omega_n^2}\right)+a_0\left(q,\omega_n\right)\right]\psi\left(p,\nu_n\right)\right\} \quad (8)$$

$$\tilde{S}\left(a_0\right)=\sum_{\omega_n}\sum_{q}\frac{1}{2}a_0\left(q,\omega_n\right)\tilde{U}^{-1}\left(q,\omega_n\right)a_0\left(-q,-\omega_n\right) \quad (9)$$

$$\tilde{U}(q,\omega_n)=\frac{g^2}{\pi v_F^{\uparrow}}\left(\frac{\omega_n^2}{\omega_n^2+\left(v_F^{\uparrow}q\right)^2}\right) \quad (10)$$

The interaction in Eq. (9) is induced by the integration of the bosonic field θ (the upper band electrons). Due to the Hubbard interaction the single particle energy of the localized band is shifted down, $\epsilon(q)\to\epsilon(q)-g^2/2\pi v_F^{\uparrow}$, $\epsilon(q)=q^2/2m$. The new gap function will be:

$$\Delta=h/2-E_F-g^2/2\pi v_F^{\uparrow}. \quad (11)$$

As a result the polarized state will exist in strong magnetic fields such that $D>0$, $\Delta>0$. In the range $D>0$ and $\Delta<0$ we obtain the hybridized state.

C. The hybridized state

The integration of the fermion field induces an effective action for this field. Keeping only second order terms in the auxiliary field we obtain the generating function $W\left(V(x,\tau)\right)$.

$$Z=\exp\left(-W\left(V^{\text{ext}}(x,\tau)\right)\right) \quad (12)$$

$$W\left(V^{\text{ext}}(x,\tau)\right)=\sum_{\omega_n}\sum_{q}\frac{1}{2\pi}V^{\text{ext}}\left(q,\omega_n\right)$$
$$\left[\frac{v_F^{\uparrow}q^2}{\left(v_F^{\uparrow}q\right)^2+\omega_n^2}+\left(1-\frac{g}{\pi v_F^{\uparrow}}\cdot\frac{\left(v_F^{\uparrow}q\right)^2}{\left(v_F^{\uparrow}q\right)^2+\omega_n^2}\right)^2\Pi\left(q,\omega_n\right)\right]V^{\text{ext}}\left(-q,-\omega_n\right). \quad (13)$$

In Eq. (13) $\Pi(q,\omega)$ is the non-interacting polarization diagram for the spin down polarized electrons. We investigate Eq. (13) at $T=0$ and $T\neq 0$.

$$\Pi\left(q,\omega_n\right) = \int \frac{dP}{2\pi}\frac{f\left(\hat{\epsilon}(p)\right) - f\left(\hat{\epsilon}(p+q)\right)}{i\omega_n - \hat{\epsilon}(p+q) + \hat{\epsilon}(q)} \tag{14}$$

Where $\tilde{\epsilon}(p) = \epsilon(p) + \Delta$, $\epsilon(p) = p^2/2m$ and $f(\epsilon + \Delta) = (\exp(\epsilon + \Delta)/T + 1)^{-1}$ is the Fermi-Dirac function. At $T \neq 0$ and large magnetic fields such that $\Delta > 0$ the polarization diagram obeys

$$v_F^{\uparrow}\pi(q,\omega_n;\ \Delta > 0) = \left(v_F^{\uparrow}/v_0^{\uparrow}\right)\exp\left(-\Delta/T\right),\ v_0^{\downarrow} = \sqrt{T/m} \tag{15}$$

For $D > 0$ and $\Delta < 0$ we have

$$v_F^{\uparrow}\pi(q,\omega_n;\ \Delta < 0) = \left(v_F^{\uparrow}/v_0^{\downarrow}\right),\ \ v_0^{\downarrow} = \sqrt{2\left(E_F + g^2/2\pi v_F - h/2\right)/m} \tag{16}$$

In order to compute the current from the generating function $W\left(V(x,\tau)\right)$, we perform the analytic continuation $\omega_n \to i\omega - x$ in Eq. (13). The current is given by:

$$I(q,\omega) = \frac{\omega}{q}\frac{\partial}{\partial V^{\text{ext}}}W\left(V^{\text{ext}}\right) \tag{17}$$

The conductance is obtained from Eq. (17) by taking the limit $\omega \to 0$, $q \to 0$. The conductance $G(T)$ in the units $e = \hbar = 1$, $1/2\pi = e^2/2\pi\hbar = e^2/h$ is

$$G(T) = e^2/h\left(1 + \left(1 - g/\pi v_F^{\uparrow}\right)^2\left(v_F^{\uparrow}/v_0^{\downarrow}\right)F(T)\right) \tag{18}$$

Where $F(T) = exp(-\Delta/T)$ for $\Delta > 0$ and $F(T) = 1$ for $\Delta < 0$ and $T \to 0$. From the Eq. (18) we observe that for large magnetic fields $\Delta \gg T$, $F(T) \to 0$.As a result $G(T) = e/h$ in agreement with the experiment. Decreasing the magnetic field, one finds that $G(T)$ increases:

$$G(T) = e^2/h\left(1 + \left(1 - g/\pi v_F^{\uparrow}\right)^2\left(v_F^{\uparrow}/\sqrt{T/m}\right)\exp(-\Delta/T)\right). \tag{19}$$

In the last part we compute the conductivity $\sigma\left(\omega,q\right)$ at $T = 0$ as a function of the Hubbard interaction g and the gap function Δ given in Eq. (11). We replace in Eq. (13,14) ω_n for $i\omega - x$ and find for the conductivity $\sigma\left(\omega,q\right)$

$$\sigma(\omega,q)=\frac{\pi e^2}{h}\left\{v_F^{\uparrow}\left(\delta\left(\omega-v_F^{\uparrow}q\right)+\delta\left(\omega+v_F^{\uparrow}q\right)\right)\right.$$
$$+\mu(-\Delta)\left[v_0^{\downarrow}\left(\delta\left(\omega-v_0^{\downarrow}q\right)+\delta\left(\omega+v_0^{\downarrow}q\right)\right)+\frac{2g}{\pi}\left(\frac{v_0^{\downarrow}v_F^{\uparrow}}{(v_F^{\uparrow})^2-(v_0^{\downarrow})^2}\right)\right.$$
$$\left.\left.\left(\delta\left(\omega-v_0^{\downarrow}q\right)+\delta\left(\omega+v_0^{\downarrow}q\right)-\delta\left(\omega-v_F^{\uparrow}q\right)-\delta\left(\omega+v_F^{\uparrow}q\right)\right)\right]\right\} \tag{20}$$

In Eq. (20) we have $\mu(-\Delta)=0$ for $\Delta>0$ and $\mu(-\Delta)=1$ for $\Delta\leq 0$, $v_0^{\downarrow}=\mu(-\Delta)\sqrt{\frac{2}{m}(-\Delta)}$ and $v_F^{\uparrow}=v_F+h$.

Using Eq. (20) we find that for an infinite system at $\omega=0$ the conductance G is given by

$$G=\begin{cases}\frac{e^2}{h}, & \Delta>0\\ \frac{2e^2}{h}, & \Delta\leq 0\end{cases} \tag{21}$$

For finite frequencies and finite samples we can have $\frac{e^2}{h}\leq G\leq\frac{2e^2}{h}$. We suggest that the quasi-plateau observed at $G=1.4e^2/h$ can be explained by assuming a strong anisotropy for the velocities $v_0^{\downarrow}\ll v_F^{\uparrow}$. This is achieved for $D=0$, $\Delta=-\frac{g^2}{2\pi v_F^{\uparrow}}$. For a finite sample of length L the conductivity at a finite frequency ω is determined by the lowest mode $v_0^{\downarrow}\frac{2\pi}{L}$, we find:

$$G=e^2/h\left[1+\frac{2g}{\pi v_F^{\uparrow}}\left(1-\left(\frac{v_0^{\downarrow}}{v_F^{\uparrow}}\right)^2\right)^{-1}\right]\simeq\frac{e^2}{h}K;\quad v_F^{\uparrow}\frac{2\pi}{L}\gg\omega\geq v_0^{\downarrow}\frac{2\pi}{L} \tag{22}$$

where $K\simeq\left[1-\frac{4g}{\pi v_F^{\uparrow}}\right]^{-1/2}>1$ is the interaction parameter for an attractive interaction generated by the electrons with opposite polarization $\left(K_F^{\uparrow}\simeq 0\right)$. This result follows, in fact, from the effective attractive interaction which originated from the repulsive interaction of the electrons with opposite spins. Eq. (22) suggests a possible qualitative explanation to the experimental value of $G=1.4e^2/h$.

To conclude, we identify the quasi-plateau observed by the Cavendish group with the strong hybridization between the "up" and "down" electrons.

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